

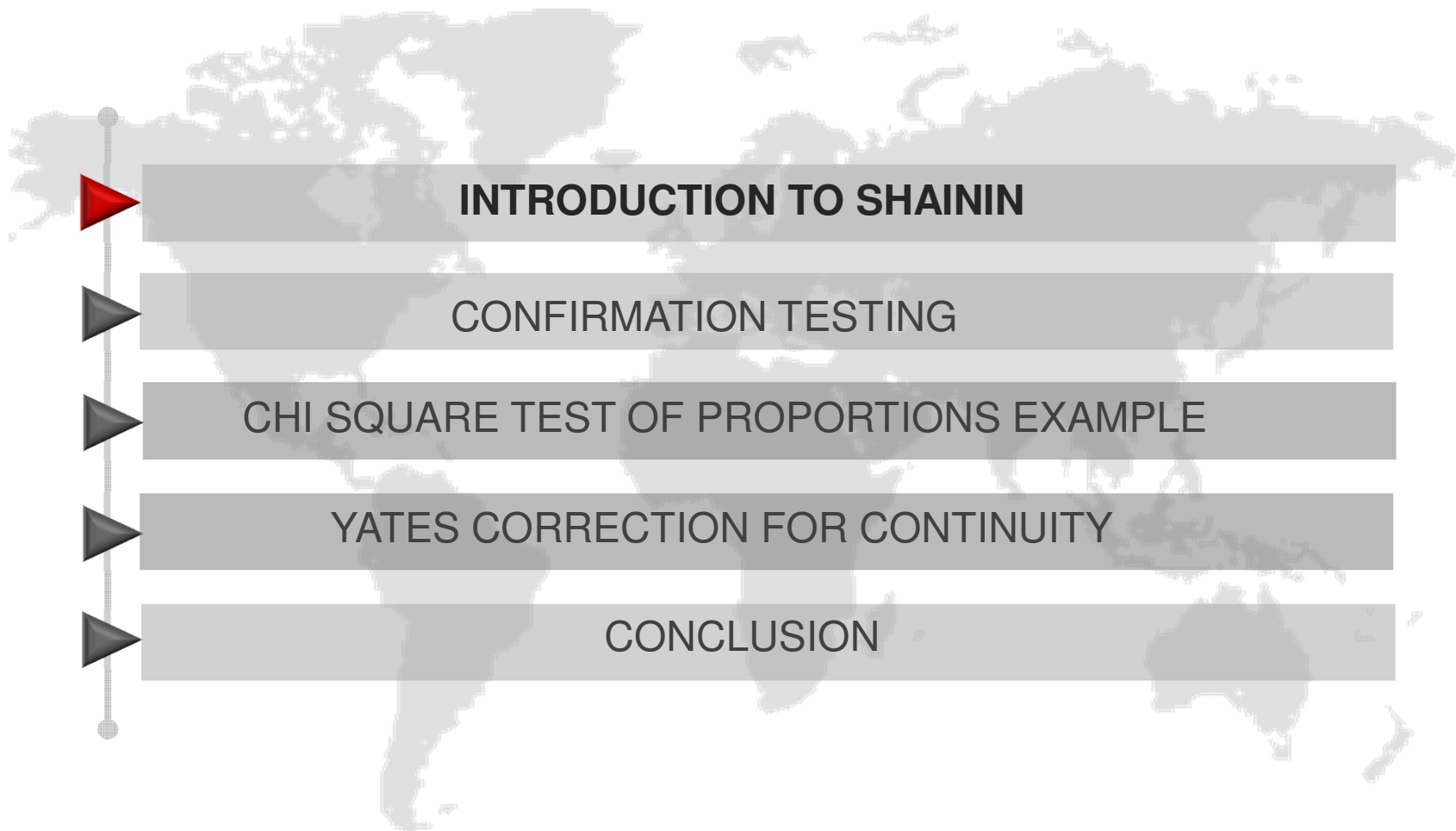


Automotive
Division
The Global Voice of Quality™

Chi Square Test of Proportions for Confirmation Testing

Craig G. Hysong
May 6, 2014

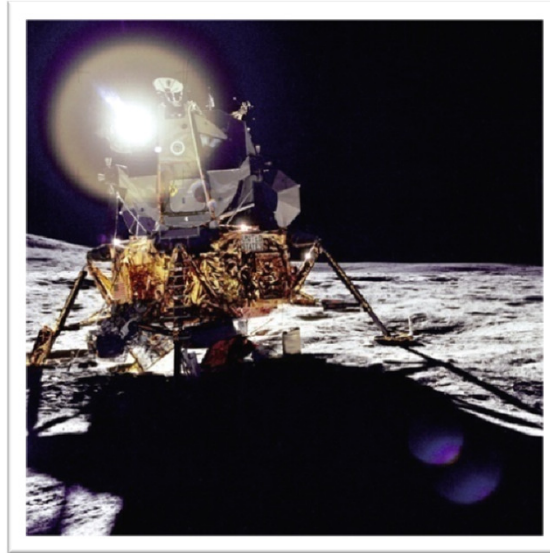
Agenda



Shainin® History



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from 1940-1945



99.99% Reliability



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Problem solving techniques originated during WWII to solve manufacturing problems in the production of Corsair F4 propellers at Hamilton Standard and its subcontractors Frigidaire, Nash-Kelvinator, and Remington-Rand.

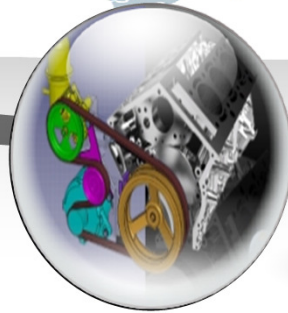
Problem solving consulting services started in 1952 and over the course of 60 years Shainin has developed over 30 statistically based techniques, working with companies representing in excess of 200 different industries.

Shainin® Offerings

Problem Solving
Services

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Shainin's Global Reach

Shainin, a global consulting company, solves and prevents critical problems in any environment.



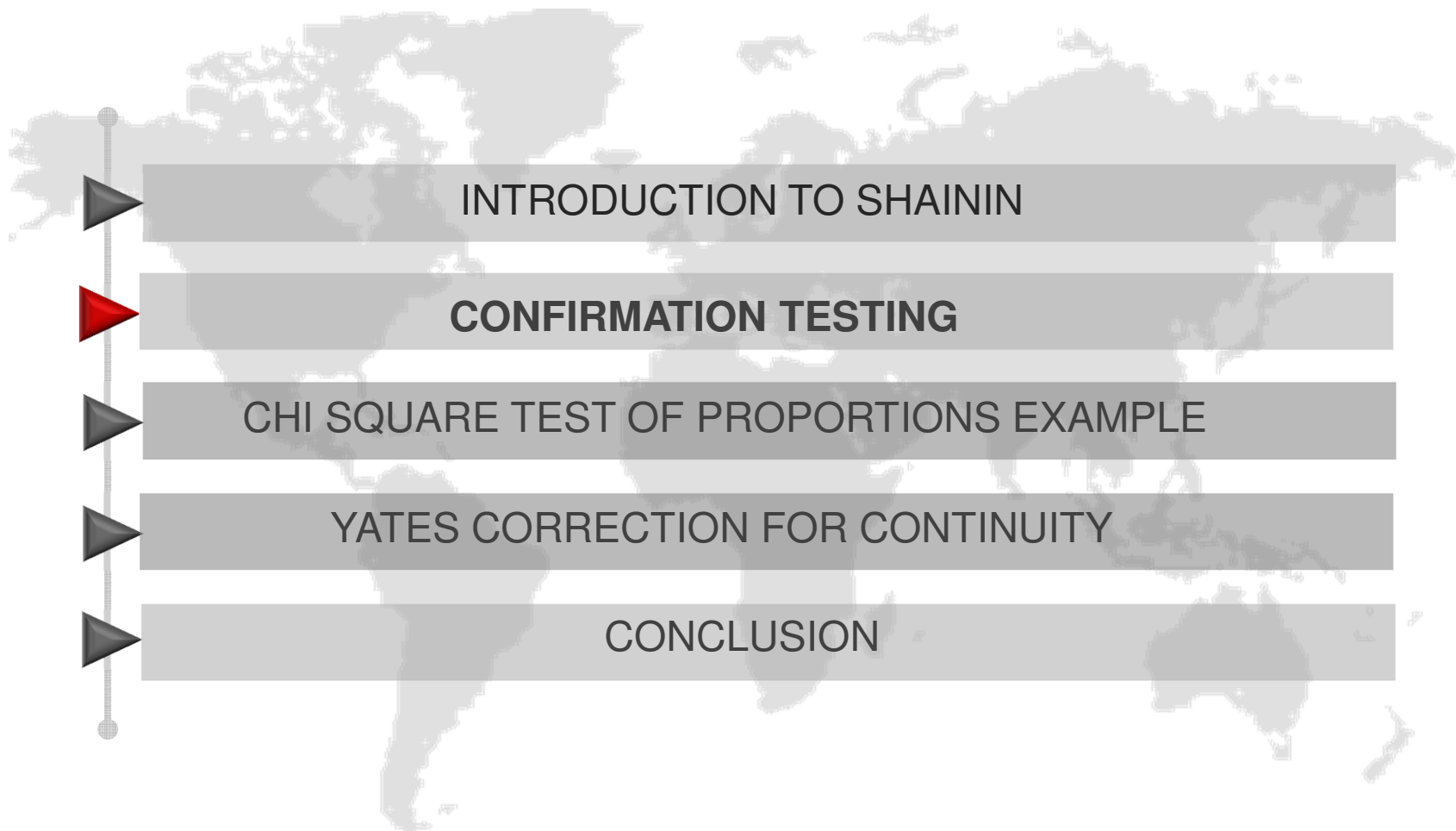
Languages:

- English
- Español
- Deutsch
- 普通话
- Français

→ Knowledge Base and Practical Experience of 1.000.000 Projects

→ Proprietary, Proven Methodologies & Tools

Agenda



B vs C Confirmation Testing

Application: Confirmation test when you have a variable response

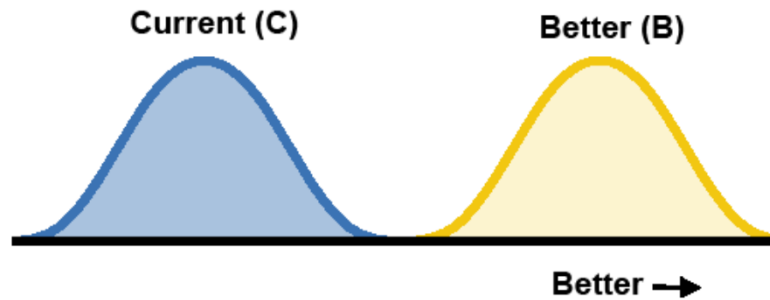
Data Set: Comparison of two unmatched groups (Proposed Better Group B to Current Group C)

Test type: Nonparametric (no assumptions made about the population distribution)

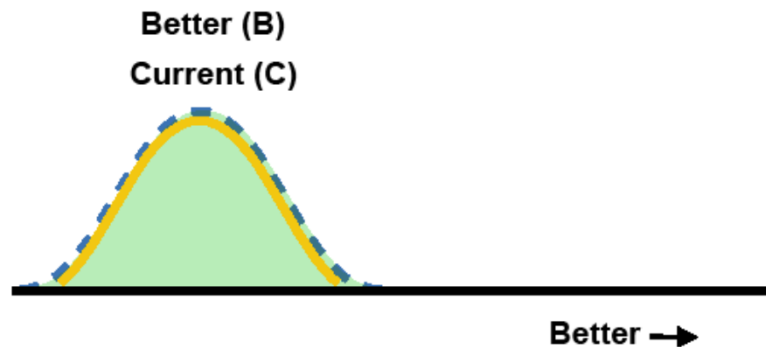
Test Statistic: Rank Order Analysis (need complete separation of groups to pass the test)

B vs C Confirmation Testing

Statistical representation of an effective change:



Statistical representation of an ineffective change:



B vs C Confirmation Testing

Consequences of a Wrong Decision		Number of Randomized Samples	
		B Samples	C Samples
Desired Confidence ~ Risk 0.999 Super Critical	0.001	3	16
		4	10
		5	8
		6	6
0.99 Critical	0.01	2	13
		3	7
		4	5
		5	4
0.95 Important	0.05	1	19
		2	5
		3	3
0.90 Moderate	0.10	1	9
		2	3

B better than C, not just different – one tailed test

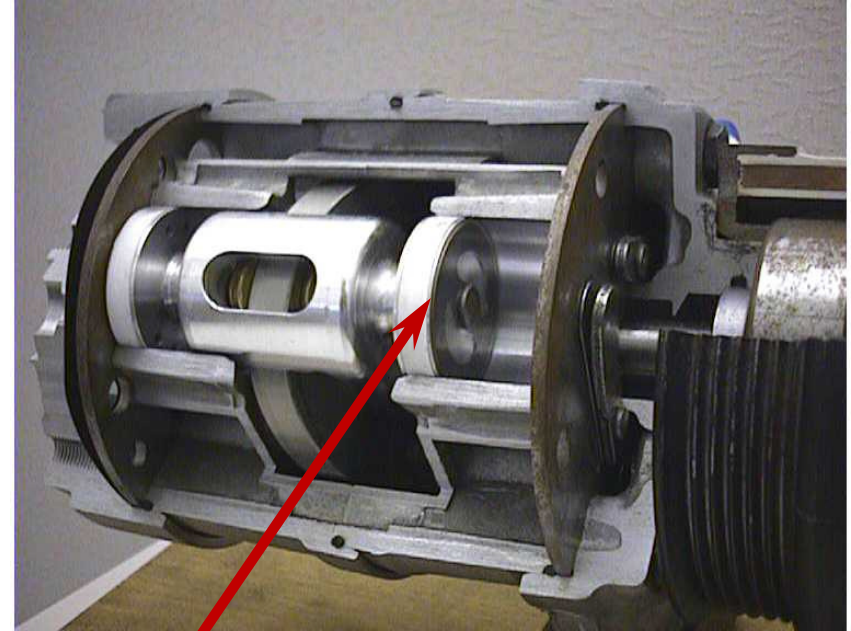
No overlap of ranks is permitted for this test – complete separation of the groups

B vs C Six Pack is the most popular test.



Compressor Broken Reed Case

- Compressors fail in the field after 21,000 miles.
- All vehicle platforms experience the failure.
- Returned compressors have extensive damage.
- The majority of warranty claims are in warmer climate.
- Warranty claims are highest in the summer months.
- Warranty costs have risen to \$20 million per year.



**New
Reed**



**Failure
Initiation**



**Failed
Reed**



Broken Reed B vs C Verifies Solution

B = Reduce suction port diameter.

C = Current suction port diameter.

Response = Cycles to failure.

Allowed risk = 5%

Required End Count = 6

Run Order		Rank Order	
Sample	Cycles to Fail	Sample	Cycles to Fail
B	+7.0M DNF	B	+7.0M DNF
B	+7.0M DNF	B	+7.0M DNF
C	3.1M FAILED	B	+7.0M DNF
B	+7.0M DNF	C	3.6M FAILED
C	3.6M FAILED	C	3.1M FAILED
C	2.8M FAILED	C	2.8M FAILED

Reducing the suction diameter will improve reed. There was a 5% risk that these test results happened by chance.

Chi-Square Test for Proportions

Application: Confirmation test when you have a binomial response (two possible outcomes)

Data Set: Comparison of two or more unmatched groups (do not care about sample size ratios)

Test type: Nonparametric (no assumptions made about the population distribution)

Test Statistic: $x^2 = \text{Sum } \frac{(\text{Observed} - \text{Expected})^2}{\text{Expected}}$

Chi-Square Test for Proportions

Expected Values: $\frac{(\text{total in row} * \text{total in column})}{\text{total population}}$

Degrees of Freedom: $(r-1)(c-1)$ where $r = \#$ rows and $c = \#$ columns

Null Hypothesis: no difference in the proportions between unmatched groups

Critical Value: χ^2 (probability, degrees of freedom) from look up table

Agenda



Setting the Stage: Condenser Braze Voids

- Our consultant receives a phone call late Friday afternoon.
- The client is missing shipments due to high scrap rates on condensers.
- Scrap is due to leaks at braze joints.
- At initial launch, scrap was not a problem since there was extra capacity.
- Ramp-up starting, so now scrap matters.
- Consultant lived 60 miles away, so he agrees to go in on Saturday morning.

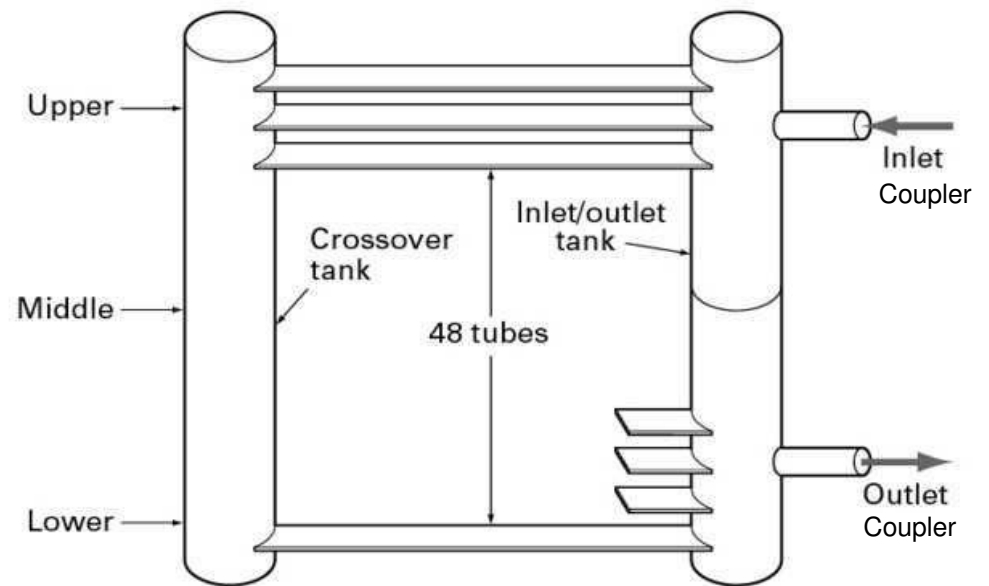
Racing the Clock

- The consultant's wife was planning a big birthday party for their daughter on Sunday afternoon with two dozen 1st graders.
- Wife tells consultant, "If your not back for the party, do not bother coming home."



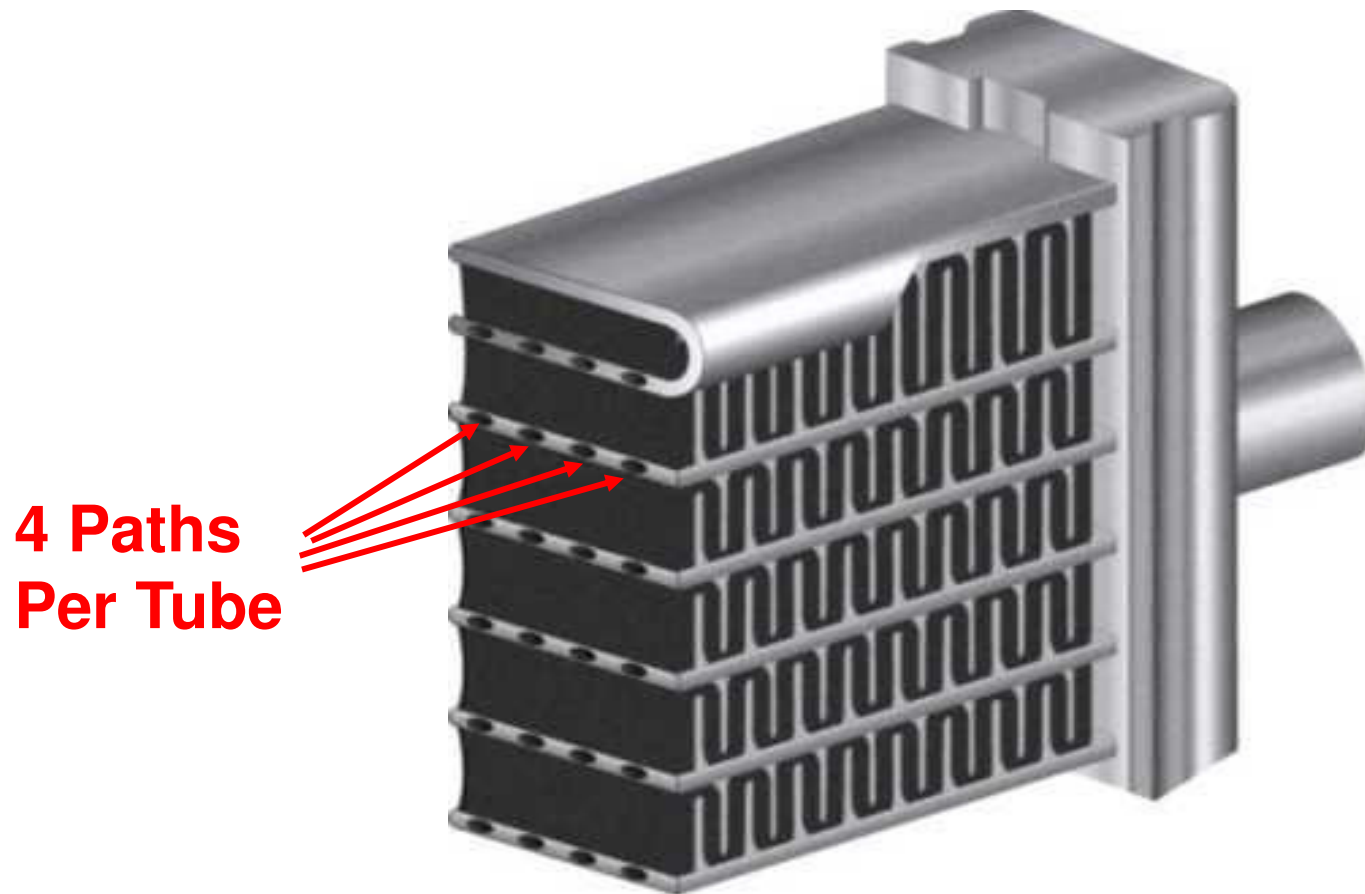
Condenser – How it Works

- Condensers are used to convert a gas to a liquid.
- Hot gas enters the inlet/outlet tank through the inlet coupler.
- The hot gas then travels through the 32 upper tubes and cools as it reaches the crossover tank.
- The cooled gas migrates to the bottom of the crossover tank and travels back to the inlet/outlet tank through the lower 16 tubes as it fully condenses to a liquid.
- The liquid exits through the outlet coupler.



Condenser – How it Works

- Each tube has four flow paths through which the fluid flows.
- Air flows around the tubes and removes heat.



Process Flow Diagram

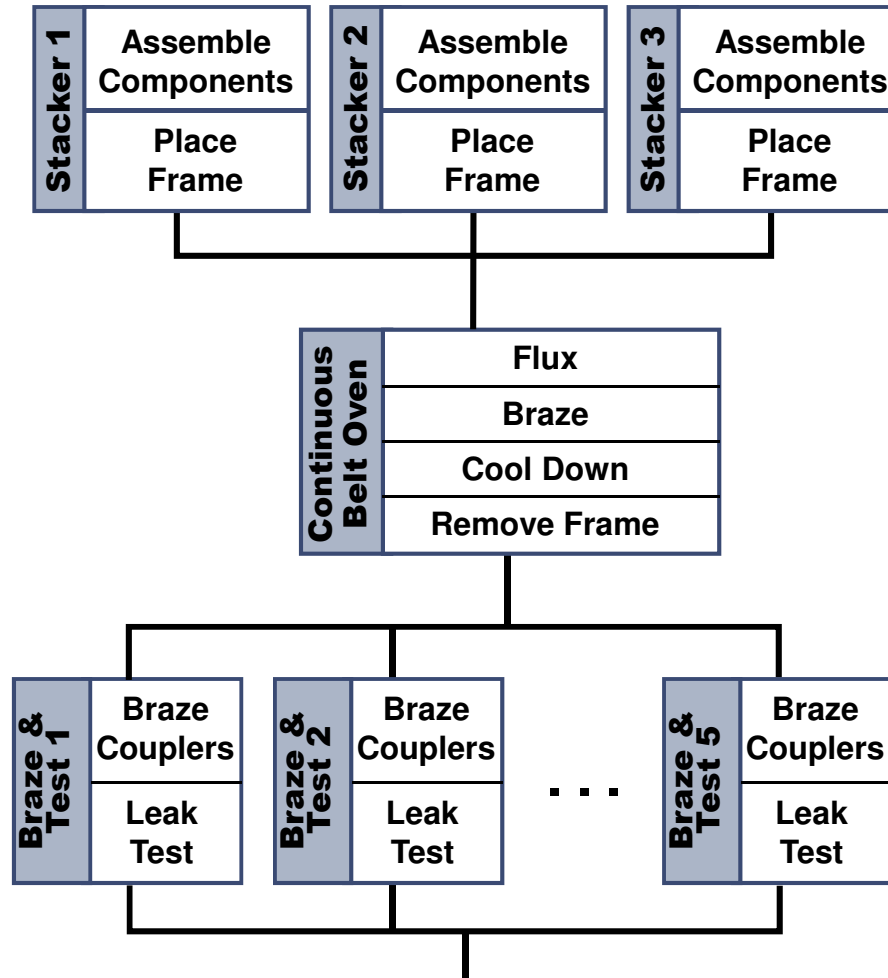
Inputs

Tanks (2)
Tubes (48)
Air Centers (47)

Unbrazed
Assembly

Brazed Assembly
(1)
Couplers (2)
Flux Rings (2)

Operation



Function

Assemble Tubes
and Air Centers to
the Tanks

Place Frame

Braze Condenser to
Seal

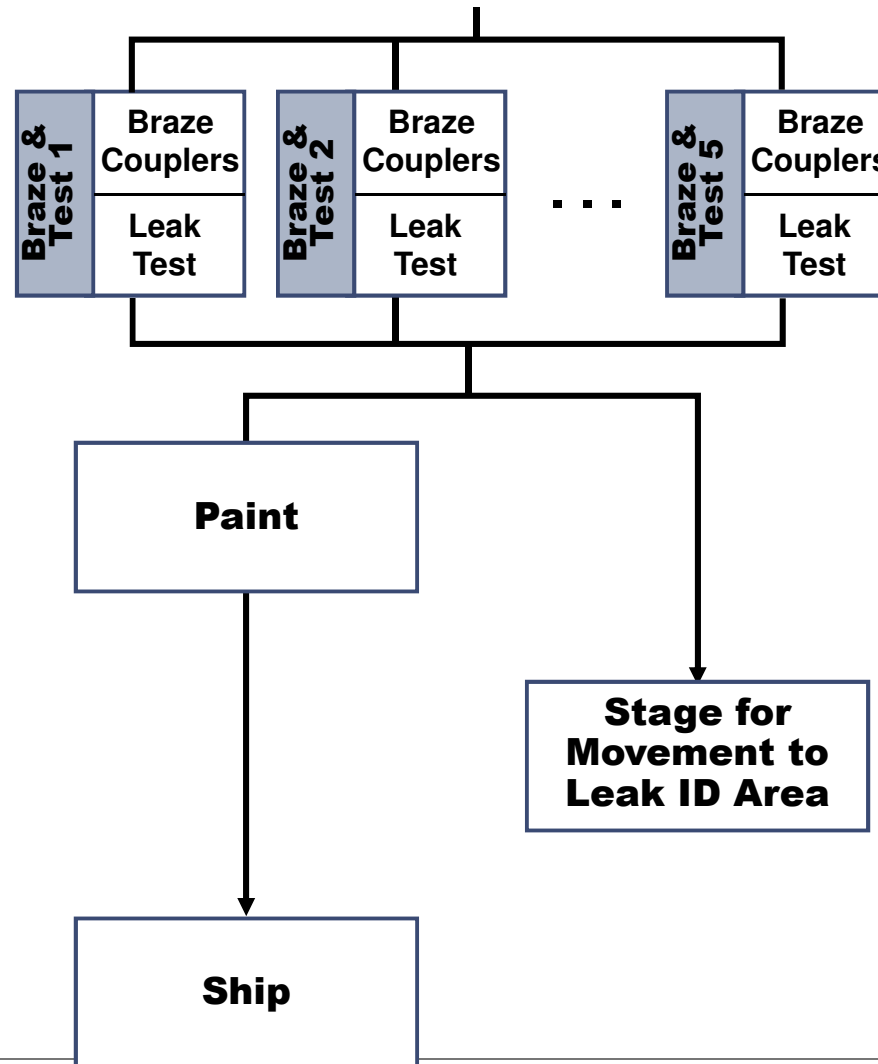
Braze Input and
Output Couplers
Leak Test Assembly

Process Flow Diagram

Inputs

Brazed Assembly (1)
Couplers (2)
Flux Rings (2)

Operation



Assemblies That Passed Leak Test

Assemblies That Failed Leak Test

Painted Assemblies

Function

Braze Input and Output Couplers
Leak Test Assembly

Apply Paint

Define Leak Area Using Bubble Test

Transport Parts

Talking to the “Experts”

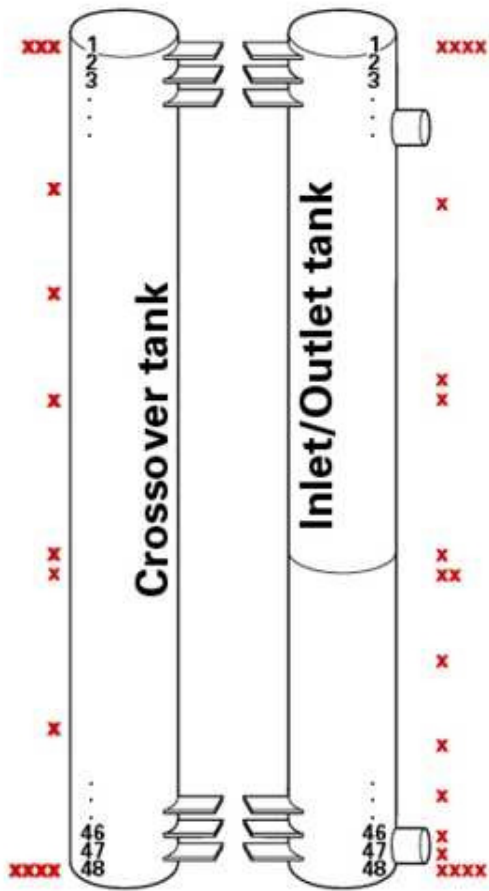
- Saturday morning all the “experts” are telling how the problem lives in the braze oven.
- The processing time for each part is 12 hours.
- There is not going to be enough time to follow parts through the process, make up for missed shipments, and get home before the party.
- The plan is to leverage the “bone pile” of existing scrapped parts and the off line Bubble Tester used to identify the leak

Saturday Morning – 8:00 AM

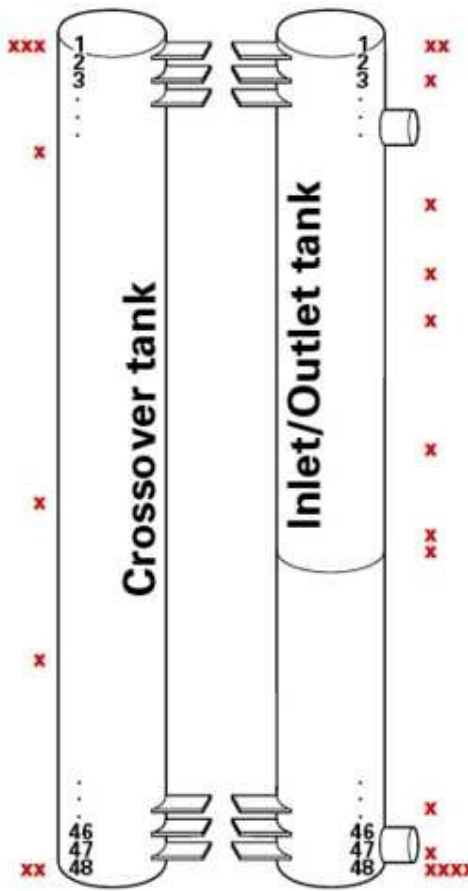


Condenser Concentration Diagram

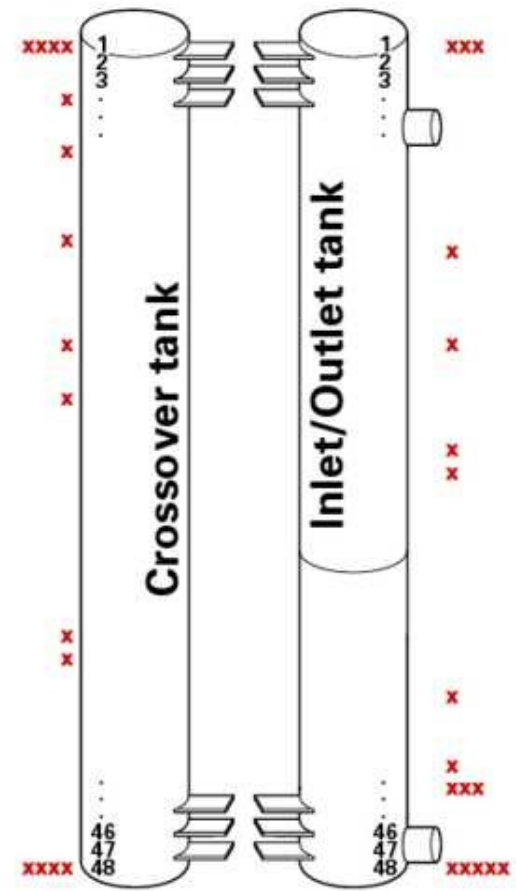
Stacker 1



Stacker 2

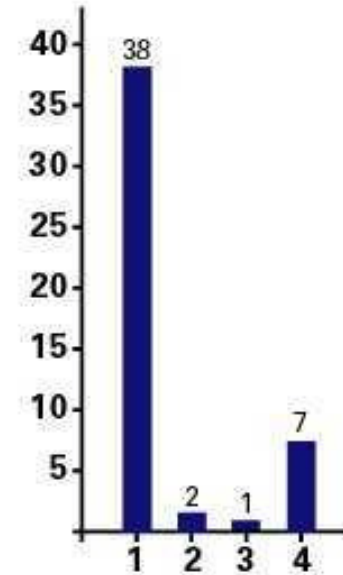
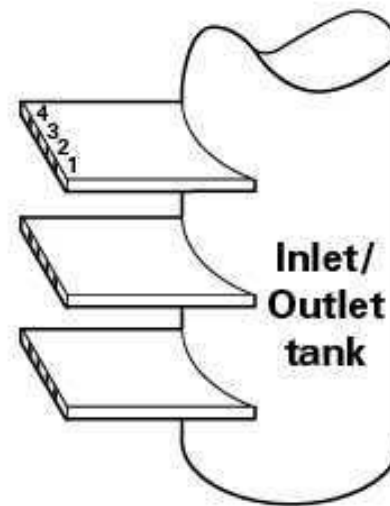
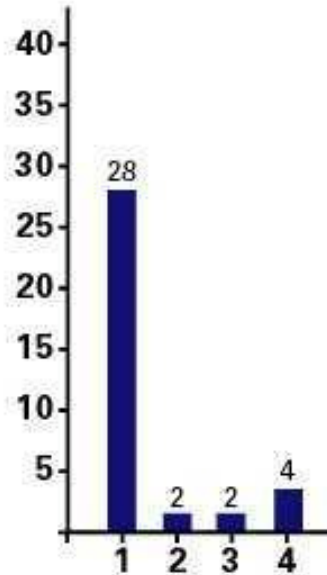
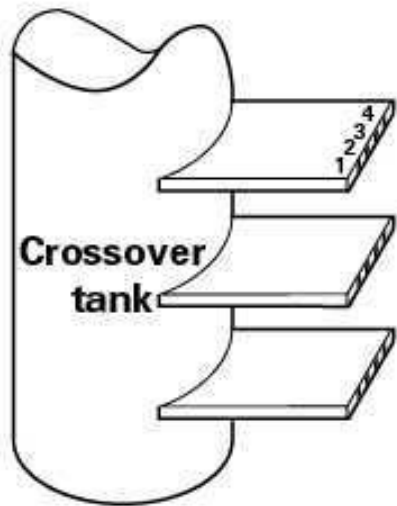


Stacker 3



Condenser Concentration Diagram

Leaks by Tube Path



Saturday Afternoon – 1:00pm



Chi Square Confirmation Test Layout

H₀ = No difference in tube #48 leak rates across the three stackers.

H_A = Shifting the stabilizing bar to contact tube #48 on stacker two has made a difference in stacker leak rates at tube #48.

Required confidence: 95%

Test: χ^2 test for proportions

Procedure: Run product across all three stackers during the C shift and Bubble Test all rejects for leaks at tube #48.

Tube 48	Stacker 1	Stacker 2	Stacker 3
No Leak			
Leak			

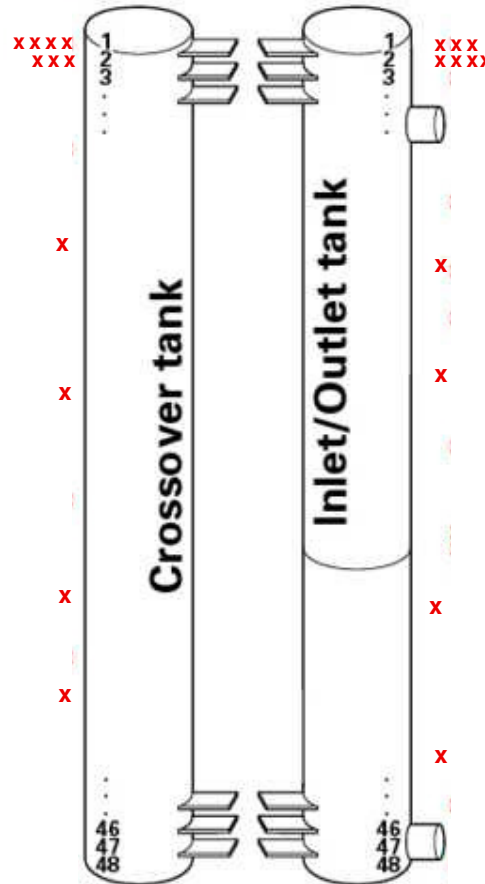
Saturday Evening – 10:00pm



Concentration Diagram

Stacker 2 with Shifted Stabilizer Bar

**No changes to
stackers one
and three!**



**Sunday Morning –
9:00 a.m.**

Chi Square Confirmation Test Results

Tube 48	Stacker 1	Stacker 2	Stacker 3
No Leak	756	891	741
Leak	144	2	121

Test Statistic:
$$x^2 = \text{Sum } \frac{(\text{Observed} - \text{Expected})^2}{\text{Expected}}$$

Chi Square Expected Test Results

Expected Values = $\frac{(\text{total in row X total in column})}{\text{total population}}$

Tube 48	Stacker 1	Stacker 2	Stacker 3	Total
No Leak	809.5	803.2	775.3	2,388
Leak	90.5	89.8	86.7	267
Total	900	893	862	2,655

Expected Value = $\frac{(2,388 \times 900)}{2,655}$

Chi Square Confirmation Test Results

Test Statistic: $x^2 = \text{Sum } \frac{(\text{Observed} - \text{Expected})^2}{\text{Expected}}$

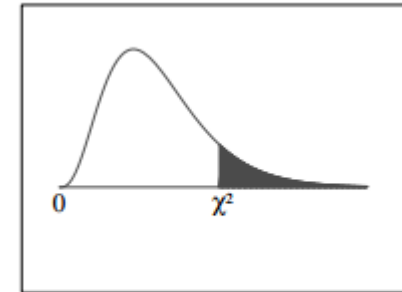
Tube 48	Stacker 1	Stacker 2	Stacker 3
No Leak	$(756-809.5)^2 / 809.5$ = 3.536	$(891-803.2)^2 / 803.2$ = 9.598	$(741-775.3)^2 / 775.3$ = 1.517
Leak	$(144-90.5)^2 / 90.5$ = 31.627	$(2-89.8)^2 / 89.8$ = 85.845	$(121-86.7)^2 / 86.7$ = 13.570

$$x^2 = 3.536 + 9.598 + 1.517 + 31.627 + 85.845 + 13.570 =$$

145.693

Chi Square Distribution Look Up Table

df	$\chi^2_{.995}$	$\chi^2_{.990}$	$\chi^2_{.975}$	$\chi^2_{.950}$	$\chi^2_{.900}$	$\chi^2_{.100}$	$\chi^2_{.050}$	$\chi^2_{.025}$	$\chi^2_{.010}$	$\chi^2_{.005}$
1	0.000	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.879	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.195	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169



The shaded area is equal to α for $\chi^2 = \chi^2_{\alpha}$.

Degrees of Freedom (n) =
 $(r-1)(c-1)$

where r = # rows and c = #
columns

$$n = (2-1) \times (3-1) = 2$$

$$\chi^2_{(0.05, 2)} = 5.991$$

Chi Square Test Result Summary

H₀ = No difference in tube #48 leak rates across the three stackers.

H_A = Shifting the stabilizing bar to contact tube #48 on stacker two has made a difference in stacker leak rates at tube #48.

Required confidence: 95%

Test: χ^2 test for proportions

Procedure: Run product across all three stackers for one shift and record all rejects for leaks at tube #48.

Tube 48	Stacker #1	Stacker #2	Stacker #3
Leak	144	2	121
No Leak	756	891	741

$$\text{d.f.} = (3-1) \times (2-1) = 2$$

$$\chi^2_{(.05, 2)} = 5.99 \text{ (critical value)}$$

$$\chi^2 = 146.4 \text{ (calculated value)}$$

Since 146.4 > 5.99 Reject H₀. There is a difference in stackers at tube #48 with 95% confidence or 5% risk.

Sunday Wrap-up

- Sunday morning meeting with executives and reported on test results until about 11:30 a.m.
- Consultant called home and reported he was on the way.



**Sunday Morning –
11:30 a.m.**



Agenda



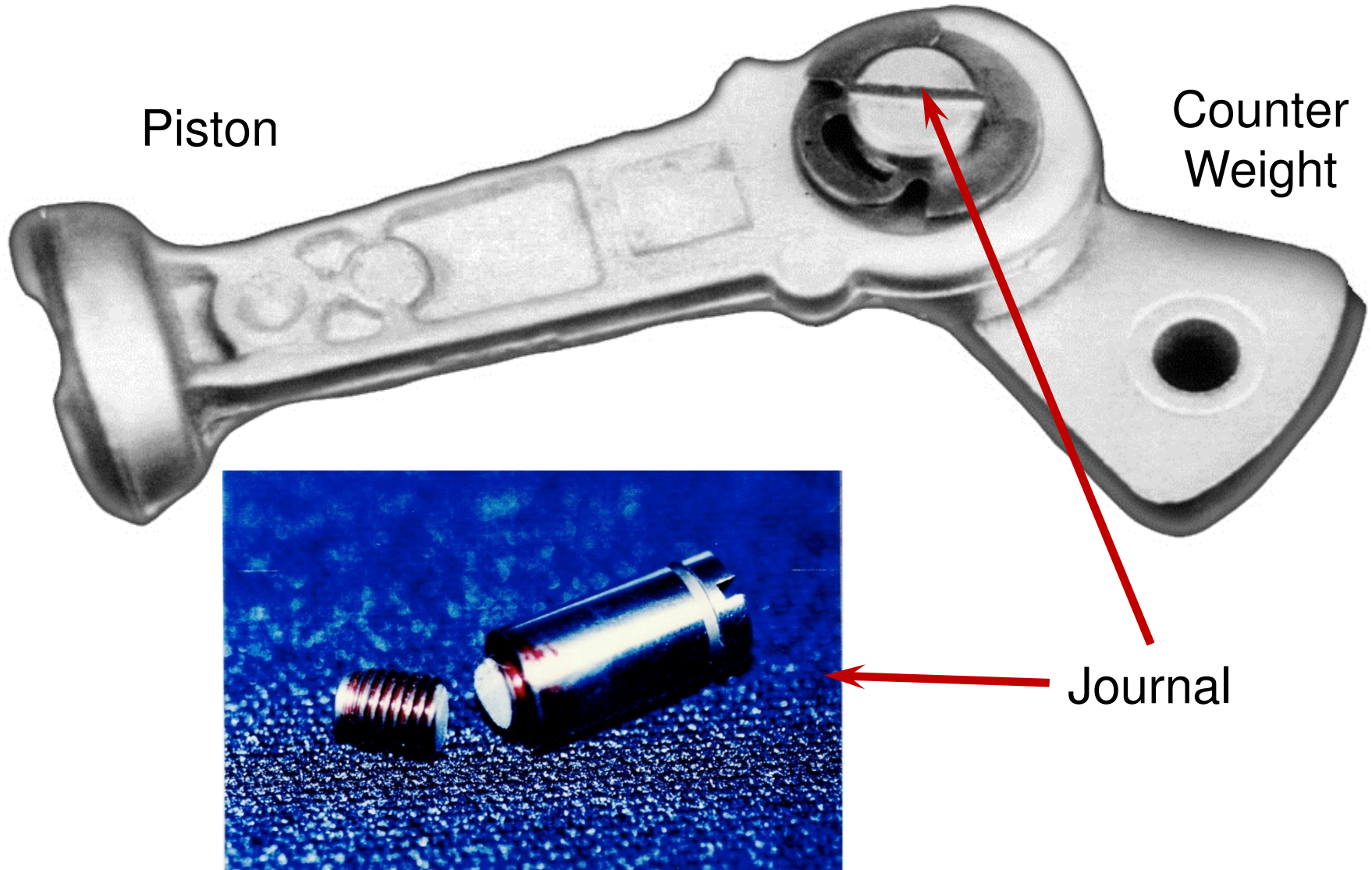
Yates Correction for Continuity

Application: When applying the Chi-Square Test for Proportions to a 2X2 table (one degree of freedom)

Test Statistic:
$$\chi^2 = \text{SUM} \frac{(| \text{Observed} - \text{Expected} | - 0.5)^2}{\text{Expected}}$$

Logic: Yate's correction prevents over estimation of statistical significance for small data sets

Air Compressor Broken Journal



Air Compressor Broken Journal

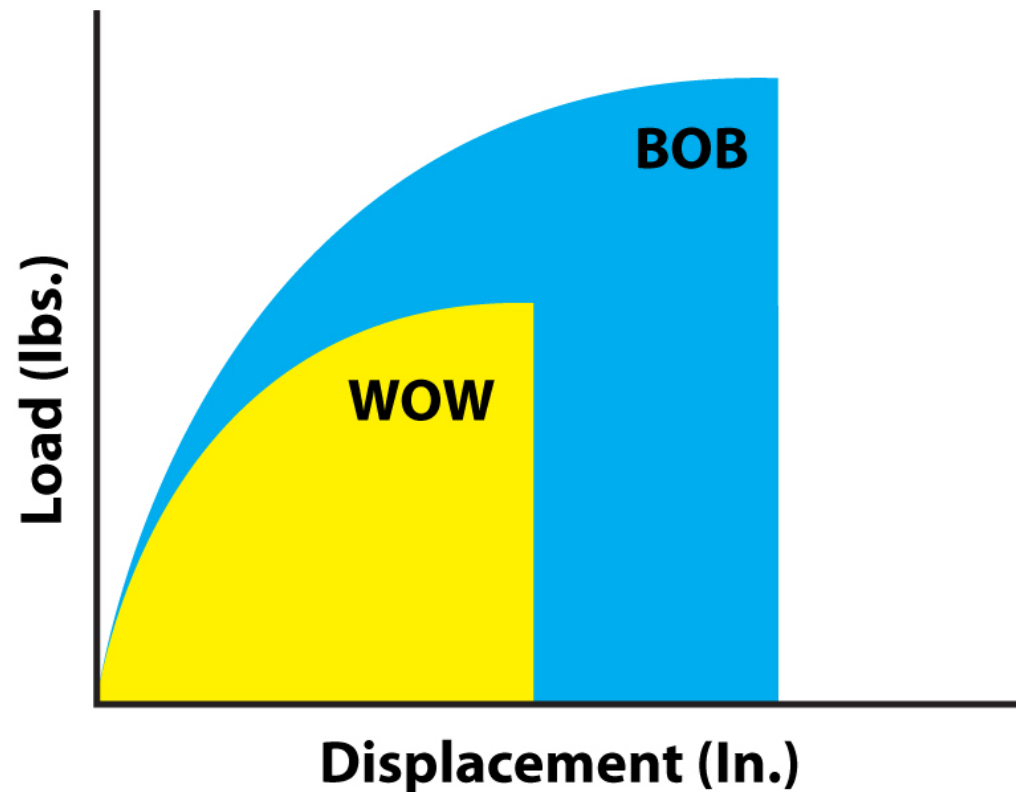
- **Very low failure rate**
- **All failures happen early in the life of the air compressor**
- **Many design and process changes impacting both strength and energy over five years to “fix” the problem**
- **Success of each “fix” determined after the latest wave of warranty data**

Air Compressor Broken Journal

Confirmed Red X as journal heat treat quench with a B vs C Six Pack Test using event to energy transform

B = quenched in salt

C = quenched in oil



Broken Journal B vs C Verifies Solution

B = Salt Quench

C = Oil Quench

Response = Tensile load in pounds to fracture

Allowed risk = 5%

Required End Count = 6

Run Order		Rank Order	
Sample	lbs. to Fracture	Sample	lbs. to Fracture
B	4,601	B	4,835
C	3,691	B	4,619
C	3,204	B	4,601
B	4,835	C	3,691
C	3,332	C	3,332
B	4,619	C	3,204

Switching to a Salt Quench will improve the strength of the Compressor Journal. There was a 5% risk that these test results happened by chance.

Air Compressor Broken Journal Case

Client's customer not comfortable with confirmation test given past claims of solving the problem combined with difficulty understanding event to energy transform.

Ran 1,500 salt quenched journals with 0 failures and 500 oil quenched journals with 3 failures.

Does the Chi-Square Test for Proportions prove with 95% confidence type of quench is the Red X root cause?

Chi Square Confirmation Test Results

Journal Results	Oil Quench	Salt Quench
Intact after customer use	497	1,500
Broken after customer use	3	0

Test Statistic: $\chi^2 = \text{SUM} \frac{(| \text{Observed} - \text{Expected} | - 0.5)^2}{\text{Expected}}$

Expected Values	Oil Quench	Salt Quench	Total
Intact after customer use	499.25	1,497.75	1,997
Broken after customer use	0.75	2.25	3
Total	500	1,500	2,000

Chi Square Confirmation Test Results

Test Statistic: $\chi^2 = \text{SUM } \frac{(| \text{Observed} - \text{Expected} | - 0.5)^2}{\text{Expected}}$

Journal Results	Oil Quench	Salt Quench
Intact after customer use	$(497-499.25 - 0.5)^2 / 499.25 = 0.006$	$(1,500-1,497.75 - 0.5)^2 / 1,497.75 = 0.002$
Broken after customer use	$(3-0.75 - 0.05)^2 / 0.75 = 4.083$	$(0-2.25 - 0.05)^2 / 2.25 = 1.361$

$$\chi^2 = 0.006 + 0.002 + 4.083 + 1.361 = \mathbf{5.452}$$

$$\chi^2_{(0.05, 1)} = \mathbf{3.841} \text{ (from slide 29)}$$

Chi Square Test Result Summary

H₀ = No difference in Journal strength (area under the curve) between a salt quench and an oil quench

H_A = There is a difference in strength between quenching in salt and oil

Required confidence: 95%

Test: χ^2 test for proportions

Procedure: Identify Journals quenched in oil and salt during manufacturing. Track Compressor 3 month warranty and identify all broken Journals by quench type.

Journal Results	Oil Quench	Salt Quench
Intact	497	1,500
Broken	3	0

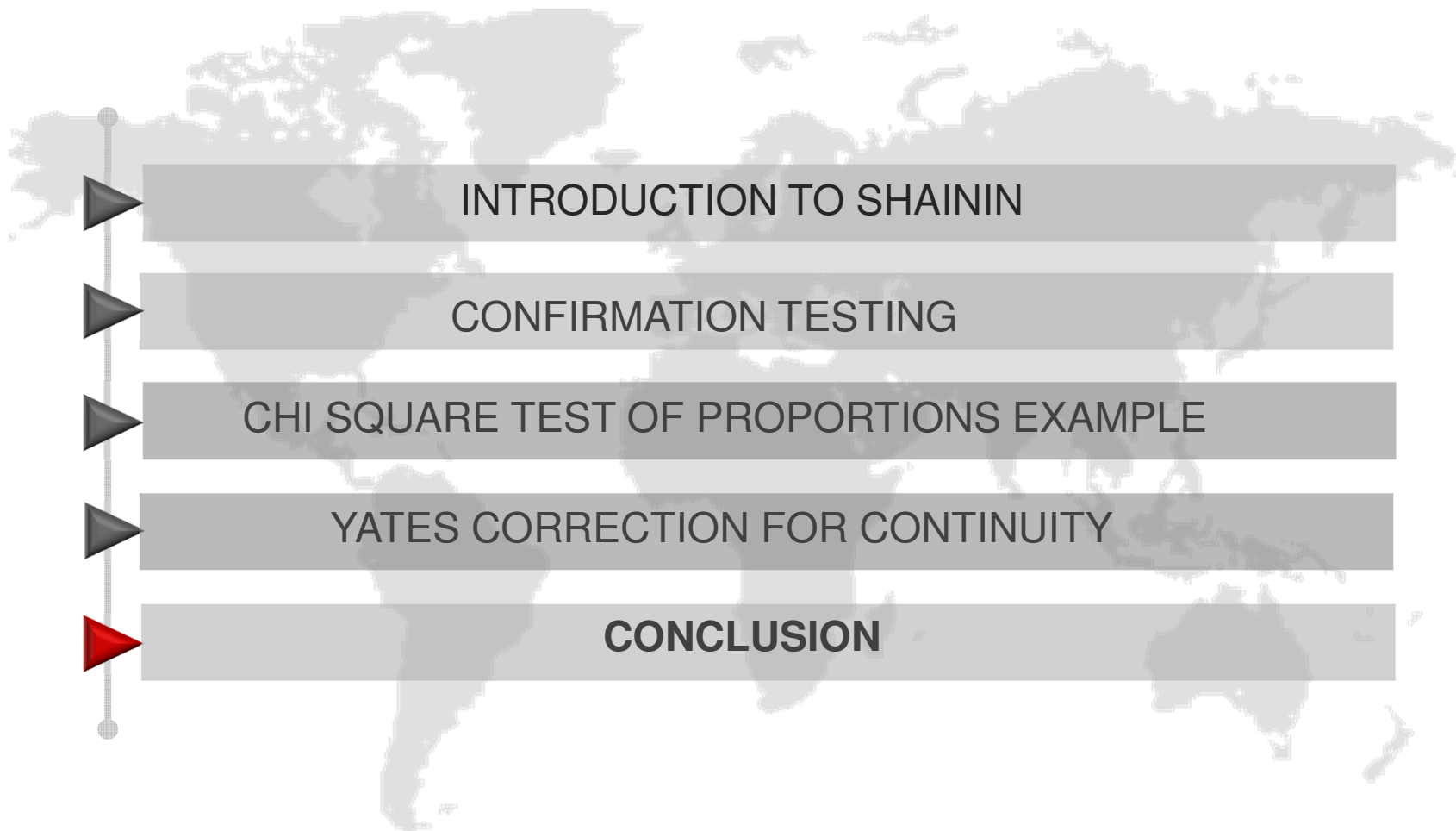
d.f. = (2-1) x (2-1) = 1 need to apply Yates Correction

$\chi^2_{(.05, 1)} = 3.841$ (critical value)

$\chi^2 = 5.452$ (calculated value)

Since 5.452 > 3.841 Reject H₀. There is a difference in Journal strength when quenched is salt verses quenched in oil at 95% confidence or 5% risk.

Agenda



Conclusions

- **Apply B vs C confirmation testing logic with variable data. Look for statistical significance** (complete separation) **and practical significance** (numerical separation).
- **Apply Chi Square confirmation testing logic with binary data. Look for statistical significance** (x^2 calculated $>$ x^2 critical value) **and practical significance** (clear distinction in data set table).
- **Apply Yates correction for continuity to be statistically conservative when the degrees of freedom (n) = 1.**



Contact & meet our experts

We help our clients to maximize
business impact.

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